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Boundary effects in super-Yang–Mills theory

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Abstract In this paper, we shall analyze a three dimensional supersymmetry theory with $\mathcal{N} = 2$ supersymmetry. We will analyze the quantization of this theory, in the presence of a boundary. The effective Lagrangian used in the path integral quantization of this theory, will be given by the sum of the gauge fixing term and the ghost term with the original classical Lagrangian. Even though the supersymmetry of this effective Lagrangian will also be broken due to the presence of a boundary, it will be demonstrated that half of the supersymmetry of this theory can be preserved by adding a boundary Lagrangian to the effective bulk Lagrangian. The supersymmetric transformation of this new boundary Lagrangian will exactly cancel the boundary term generated from the supersymmetric transformation of the effective bulk Lagrangian. We will analyze the Slavnov–Taylor identity for this $\mathcal{N} = 2$ Yang–Mills theory with a boundary.

1 Introduction

As any gauge theory contains unphysical gauge degrees of freedom, and it is not possible quantize this theory without removing these unphysical degrees of freedom. This is achieved by fixing a gauge, and the gauge fixing is incorporated at a quantum level by adding a gauge fixing term to the original Lagrangian. We also need to add a ghost term corresponding to this gauge fixing term to the original Lagrangian. This new effective Lagrangian obtained from a sum of the original classical Lagrangian with the gauge fixing and the ghost terms is invariant under the BRST transformations [1,2]. The BRST symmetry has been studied for various different gauges [3–7], and it has been applied for analyzing various aspects of different supersymmetric theories [8–12].

The BRST symmetry has also been used in analyzing ghost–anti-ghost condensation [13–17]. Furthermore, such ghost–anti-ghost condensation has been proposed as the mass providing mechanism of the off-diagonal gluons and off-diagonal ghosts in the Yang–Mills theory [18,19]. This analysis has been performed using the maximal Abelian gauge. Evidence for infrared Abelian dominance has also been provided by this mechanism [20], thereby justifying the dual superconductor picture [21–23] of the QCD vacuum. This has been used in explaining quark confinement [19–26]. It may be noted that interesting consequences of the breaking of BRST symmetry have also been discussed [13–30].

The action for most renormalizable quantum field theories, including supersymmetric theories, is at most quadratic in the derivatives. So, the supersymmetric variation of such an action produces a total derivative term. In the absence of a boundary this total derivative term vanishes. However, in the presence of a boundary, boundary contributions arise due to such a total derivative term. This breaks the supersymmetry of a supersymmetric theory in the presence of a boundary. It may be noted that the translational invariance of any theory is broken by the presence of a boundary. The breaking of the translational invariance in a supersymmetric theory also breaks the supersymmetry of that theory. However, it is possible to retain some on-shell supersymmetry by imposing suitable boundary conditions [31,32]. The supersymmetry of a theory generates various constraints on the possible boundary conditions [33–37].

Even though some on-shell supersymmetry can be retained by imposing boundary conditions, the off-shell supersymmetry is still broken. This is because these boundary conditions are only imposed on the on-shell field. It is important to preserve the off-shell supersymmetry of a theory. This is because the path integral formalism uses off-shell fields, and most supersymmetric theories are quantized using a path inte-

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gral formalism. So, it is important to preserve the off-shell supersymmetry for a theory. It is possible to preserve half the off-shell supersymmetry of a theory by modifying the original action of the theory. This has been done by the addition of new boundary terms to the original bulk action. The boundary contribution generated from the supersymmetric variation of the original bulk action are exactly canceled by the supersymmetric variation of these new boundary terms. This has been studied for a three dimensional theory with $\mathcal{N} = 1$ supersymmetry [38]. Furthermore, this procedure has been used for analyzing Chern–Simons-matter theories in the presence of a boundary [8,39–41]. It may be noted that an additional boundary term is also generated from the gauge transformation of Chern–Simons-matter theories in the presence of a boundary. So, additional boundary degrees of freedom are needed to preserve the gauge invariance of a Chern–Simons-matter theory in the presence of a boundary. This is because the boundary contribution generated from the gauge transformation of the bulk action are exactly canceled by the gauge transformation of these new boundary degrees of freedom.

A non-anticommutative deformation of supersymmetric theories has also been studied using this off-shell formalism [42]. This has been done for a theory with $\mathcal{N} = 2$ supersymmetric theory in three dimensions. In this analysis, half the supersymmetry of such a supersymmetric theory was broken by imposing non-anticommutativity. Then by suitably combining the boundary effects with non-anticommutativity, a theory with $\mathcal{N} = 1/2$ supersymmetry was constructed. A three dimensional super-Yang–Mills theory has also been coupled to background flux in the presence of a boundary [43]. In this paper, we shall make an analysis of gauge theory with $\mathcal{N} = 2$ supersymmetry in the presence of a boundary. It is important to perform such an analysis to demonstrate the preservation of half the supersymmetry for a gauge theory at the quantum level. So, we will analyze this theory using the quantum fluctuations around a fixed background. We will analyze the BRST symmetry of such a theory, by analyzing the fields as a sum of the classical background fields and quantum fluctuations around such classical fields. We will also analyze the Slavnov–Taylor identity for such a theory.

2 Supersymmetric gauge theory

In this section, we will review the construction of a three dimensional supersymmetric gauge theory in the presence of a boundary [8,38–42]. We define two fermionic coordinates, $\theta_{1a} = (\theta_{11}, \theta_{12})$ and $\theta_{2a} = (\theta_{2a}, \theta_{22})$. Now we can also define $(\gamma^\mu\theta_1)_a = (\gamma^\mu)_a^b\theta_{1a}$ and $(\gamma^\mu\theta_2)_a = (\gamma^\mu)_a^b\theta_{2a}$. The raising and lowering of the spinor indices occurs as $\theta_1^a = C^{ba}\theta_{1b}$, $\theta_{1a} = \theta_1^b C_{ab}$, and $\theta_2^a = C^{ba}\theta_{2b}$, $\theta_{2a} = \theta_2^b C_{ab}$. Here $C^{ab} = -C^{ba}$, $C_{ab} = -C_{ba}$, and $C^{ba}C_{cb} = \delta_c^a$. We also have $(\gamma^\mu)_{ab} = (\gamma^\mu)_a^c C_{cb} = (\gamma^\mu)_{ba}$. A $\mathcal{N} = 2$ supersymmetric theory in three dimensions can be parameterized by two supercharges,

$$Q_{1a} = \partial_{1a} - (\gamma^\mu\theta_1)_a\partial_\mu, \quad Q_{2a} = \partial_{2a} - (\gamma^\mu\theta_2)_a\partial_\mu. \quad (1)$$

These supercharges satisfy,

$$\{Q_{1a}, Q_{1b}\} = 2\gamma^\mu_{ab}\partial_\mu, \{Q_{2a}, Q_{2b}\} = 2\gamma^\mu_{ab}\partial_\mu,$$
$$\{Q_{1a}, Q_{2b}\} = 0. \quad (2)$$

Now we define superderivatives by

$$D_{1a} = \partial_{1a} + (\gamma^\mu\theta_1)_a\partial_\mu, \quad D_{2a} = \partial_{2a} + (\gamma^\mu\theta_2)_a\partial_\mu. \quad (3)$$

These superderivatives commute with the generators of $\mathcal{N} = 2$ supersymmetry, $\{Q_{1a}, D_{1b}\} = \{Q_{1a}, D_{2b}\} = 0$ and $\{Q_{2a}, D_{1b}\} = \{Q_{2a}, D_{2b}\} = 0$. These superderivatives also satisfy

$$\{D_{1a}, D_{1b}\} = -2\gamma^\mu_{ab}\partial_\mu, \{D_{2a}, D_{2b}\} = -2\gamma^\mu_{ab}\partial_\mu,$$
$$\{D_{1a}, D_{2b}\} = 0. \quad (4)$$

We can also define gauge valued spinor superfields $\Gamma_{1a} = \Gamma^A_{1a}(\theta_1)T_A$ and $\Gamma_{2a} = \Gamma^A_{2a}(\theta_2)T_A$, where $[T_A, T_B] = if^C_{AB}T_C$. Now we can define covariant derivatives with these fields by

$$\nabla_{1a} = D_a - i\Gamma_{1a}, \quad \nabla_{2a} = D_a - i\Gamma_{2a}. \quad (5)$$

These fields transform under the gauge transformation as $\Gamma_{1a} \to iu\nabla_{1a}u^{-1}$, and $\Gamma_{2a} \to iu\nabla_{2a}u^{-1}$ [44]. We can also construct the field strengths as follows:

$$W_{1a} = \frac{1}{2}D_1^b D_{1a}\Gamma_{1b} - \frac{i}{2}\{\Gamma_1^b, D_{1b}\Gamma_{1a}\} - \frac{1}{6}[\Gamma_1^b, \{\Gamma_{1b}, \Gamma_{1a}\}],$$

$$W_{2a} = \frac{1}{2}D_2^b D_{2a}\Gamma_{2b} - \frac{i}{2}\{\Gamma_2^b, D_{2b}\Gamma_{2a}\} - \frac{1}{6}[\Gamma_1^2, \{\Gamma_{2b}, \Gamma_{2a}\}]. \quad (6)$$

These field strengths transform as $W_{1a} \to uW_{1a}u^{-1}$, and $W_{2a} \to uW_{2a}u^{-1}$. We can write the action for super-Yang–Mills theory as

$$\mathcal{L} = D_1^2[W_1^a W_{1a}]_{\theta_1=0} + D_2^2[W_2^a W_{2a}]_{\theta_2=0}. \quad (7)$$

In the presence of a boundary the supersymmetry is broken. However, half of the supersymmetry of the original theory can be preserved by either adding or subtracting a boundary term to the original Lagrangian [38]. We now define a boundary along x_3 direction. Thus, we can define the boundary fields as fields restricted to the boundary, and we can also construct boundary Lagrangian from such fields. We can define $\mathcal{L}_{1b}$ and $\mathcal{L}_{2b}$ to be such boundary Lagrangian constructed from the boundary fields. Now this boundary Lagrangian can be added or subtracted from the bulk Lagrangian with $\mathcal{N} = 2$ supersymmetry. It is possible to choose this boundary Lagrangian such that $\mathcal{L} \pm \mathcal{L}_{1b}$ preserves the supersymmetry generated by $\epsilon^{1\mp}Q_{1\pm}$, and

$\mathcal{L} \pm \mathcal{L}_{2b}$ preserves the supersymmetry generated by $\epsilon^{2\mp} Q_{2\pm}$ [42]. Here the projection operators $P_\pm = (1 \pm \gamma^3)/2$ have been used to obtain these projections of the supercharges. Now as the original Lagrangian $\mathcal{L} = D_1^2[\Omega_1(\theta_1)]_{\theta_1=0}$ and $\mathcal{L} = D_2^2[\Omega_2(\theta_2)]_{\theta_2=0}$, the boundary terms can be written as $\mathcal{L}_{1b} = \partial_3[\Omega_1(\theta_1)]_{\theta_1=0}$ and $\mathcal{L}_{2b} = \partial_3[\Omega_2(\theta_2)]_{\theta_2=0}$ [39]. It is not possible to simultaneously preserve both the supersymmetry generated by $\epsilon^{1-} Q_{1+}$ and $\epsilon^{1+} Q_{1-}$, or $\epsilon^{2-} Q_{2+}$ and $\epsilon^{2+} Q_{2-}$, in the presence of a boundary. However, in the presence of a boundary, we can construct the Lagrangian which preserves the supersymmetry generated by $\epsilon^{1\mp} Q_{1\pm}$ and $\epsilon^{2\mp} Q_{2\pm}$. We can write the Lagrangian for super-Yang–Mills theory which preserves various supersymmetries as [42],

$$
\begin{aligned}
\mathcal{L}^{1-2-} &= (D_1^2 - \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 - \partial_3)[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1-2+} &= (D_1^2 - \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 + \partial_3)[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1+2-} &= (D_1^2 + \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 - \partial_3)[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1+2+} &= (D_1^2 + \partial_3)[W_1^a W_{1a}]_{\theta_1=0} + (D_2^2 + \partial_3)[W_2^a W_{2a}]_{\theta_2=0}.
\end{aligned} \tag{8}
$$

3 BRST symmetry

In this section, we will study the effective Lagrangian obtained by the sum of the gauge fixing term and the ghost term with the modified super-Yang–Mills Lagrangian in the Lorenz gauge. The Lorenz gauge fixing conditions can be incorporated in the modified super-Yang–Mills Lagrangian at a quantum level by adding the following gauge fixing term:

$$
\begin{aligned}
\mathcal{L}_{gf}^{1+2+} &= (D_1^2 + \partial_3)\left[b_1(D_1^a \Gamma_{1a}) + \frac{\alpha}{2} b_1^2\right]_{\theta_1=0} \\
&\quad + (D_2^2 + \partial_3)\left[b_2(D_2^a \Gamma_{2a}) + \frac{\alpha}{2} b^2\right]_{\theta_2=0}, \\
\mathcal{L}_{gf}^{1-2-} &= (D_1^2 - \partial_3)\left[b_1(D_1^a \Gamma_{1a}) + \frac{\alpha}{2} b_1^2\right]_{\theta_1=0} \\
&\quad + (D_2^2 - \partial_3)\left[b_2(D_2^a \Gamma_{2a}) + \frac{\alpha}{2} b^2\right]_{\theta_2=0}, \\
\mathcal{L}_{gf}^{1+2-} &= (D_1^2 + \partial_3)\left[b_1(D_1^a \Gamma_{1a}) + \frac{\alpha}{2} b_1^2\right]_{\theta_1=0} \\
&\quad + (D_2^2 - \partial_3)\left[b_2(D_2^a \Gamma_{2a}) + \frac{\alpha}{2} b^2\right]_{\theta_2=0}, \\
\mathcal{L}_{gf}^{1-2+} &= (D_1^2 - \partial_3)\left[b_1(D_1^a \Gamma_{1a}) + \frac{\alpha}{2} b_1^2\right]_{\theta_1=0} \\
&\quad + (D_2^2 + \partial_3)\left[b_2(D_2^a \Gamma_{2a}) + \frac{\alpha}{2} b^2\right]_{\theta_2=0}.
\end{aligned} \tag{9}
$$

where b_1 and b_2 are Nakanishi–Lautrup type auxiliary fields. The ghost term corresponding to this gauge fixing term can be written as

$$
\begin{aligned}
\mathcal{L}_{gh}^{1+2+} &= (D_1^2 + \partial_3)[\bar{c}_1 D_1^a \nabla_{1a} c_1]_{\theta_1=0} \\
&\quad + (D_2^2 + \partial_3)[\bar{c}_2 D_2^a \nabla_{2a} c_2]_{\theta_2=0}, \\
\mathcal{L}_{gh}^{1-2-} &= (D_1^2 - \partial_3)[\bar{c}_1 D_1^a \nabla_{1a} c_1]_{\theta_1=0} \\
&\quad + (D_2^2 - \partial_3)[\bar{c}_2 D_2^a \nabla_{2a} c_2]_{\theta_2=0}, \\
\mathcal{L}_{gh}^{1+2-} &= (D_1^2 + \partial_3)[\bar{c}_1 D_1^a \nabla_{1a} c_1]_{\theta_1=0} \\
&\quad + (D_2^2 - \partial_3)[\bar{c}_2 D_2^a \nabla_{2a} c_2]_{\theta_2=0}, \\
\mathcal{L}_{gh}^{1-2+} &= (D_1^2 - \partial_3)[\bar{c}_1 D_1^a \nabla_{1a} c_1]_{\theta_1=0} \\
&\quad + (D_2^2 + \partial_3)[\bar{c}_2 D_2^a \nabla_{2a} c_2]_{\theta_2=0}
\end{aligned} \tag{10}
$$

where c_1, c_2 are the ghost fields and $\bar{c}_1, \bar{c}$ are the anti-ghost fields. Now we can define $\mathcal{L}_g^{1\pm2\pm}$ as

$$
\mathcal{L}_g^{1+2+} = \mathcal{L}_{gf}^{1\pm2\pm} + \mathcal{L}_{gh}^{1\pm2\pm}. \tag{11}
$$

The effective Lagrangian $\mathcal{L}^{1\pm2\pm} = \mathcal{L}^{1\pm2\pm} + \mathcal{L}_g^{1\pm2\pm}$, which is given by the sum of the ghost and the gauge fixing terms with modified super-Yang–Mills Lagrangian, is invariant under the following BRST transformations:

$$
\begin{aligned}
& s_b\, \Gamma_{1a} = \nabla_{1a} c_1, \quad s_b\, \Gamma_{2a} = \nabla_{2a} c_2, \\
& s_b\, c_1 = -\frac{1}{2}[c_1, c_1], \quad s_b\, c_2 = -\frac{1}{2}[c_2, c_2], \\
& s_b\, \bar{c}_1 = b_1, \quad s_b\, \bar{c}_2 = b_2, \\
& s_b\, b_1 = 0, \quad s_b\, b_2 = 0,
\end{aligned} \tag{12}
$$

This is because modified super-Yang–Mills Lagrangian is BRST invariant, and the sum of the gauge fixing and ghost terms can be expressed as

$$
\begin{aligned}
\mathcal{L}_g^{1+2+} &= s_b(D_1^2 + \partial_3)\left[\bar{c}_1 D_1^a \Gamma_{1a} + \frac{\alpha}{2} \bar{c}_1 b_1\right]_{\theta_1=0} \\
&\quad + s_b(D_2^2 + \partial_3)\left[\bar{c}_2 D_2^a \Gamma_{2a} + \frac{\alpha}{2} \bar{c}_2 b_2\right]_{\theta_2=0}, \\
\mathcal{L}_g^{1-2-} &= s_b(D_1^2 - \partial_3)\left[\bar{c}_1 D_1^a \Gamma_{1a} + \frac{\alpha}{2} \bar{c}_1 b_1\right]_{\theta_1=0} \\
&\quad + s_b(D_2^2 - \partial_3)\left[\bar{c}_2 D_2^a \Gamma_{2a} + \frac{\alpha}{2} \bar{c}_2 b_2\right]_{\theta_2=0}, \\
\mathcal{L}_g^{1+2-} &= s_b(D_1^2 + \partial_3)\left[\bar{c}_1 D_1^a \Gamma_{1a} + \frac{\alpha}{2} \bar{c}_1 b_1\right]_{\theta_1=0} \\
&\quad + s_b(D_2^2 - \partial_3)\left[\bar{c}_2 D_2^a \Gamma_{2a} + \frac{\alpha}{2} \bar{c}_2 b_2\right]_{\theta_2=0}, \\
\mathcal{L}_g^{1-2+} &= s_b(D_1^2 - \partial_3)\left[\bar{c}_1 D_1^a \Gamma_{1a} + \frac{\alpha}{2} \bar{c}_1 b_1\right]_{\theta_1=0} \\
&\quad + s_b(D_2^2 + \partial_3)\left[\bar{c}_2 D_2^a \Gamma_{2a} + \frac{\alpha}{2} \bar{c}_2 b_2\right]_{\theta_2=0}.
\end{aligned} \tag{13}
$$

Now as $s_b^2 = 0$, the sum of the gauge fixing and ghost terms is also invariant under the BRST transformations.

It is possible to analyze this theory with fixed background fields, and quantum fluctuations around these fields. We can obtain the BRST symmetry of such a theory. The Lagrangian is expressed in terms of classical background fields and quantum fluctuations around these fields,

$$\begin{aligned}&\mathcal{L}^{1\pm2+\pm}(\Gamma_1,\Gamma_2)+\mathcal{L}_g^{1\pm2+\pm}(\Gamma_1,\Gamma_2,c_1,c_2,\bar{c}_1,\bar{c}_2,b_1,b_2)\\&\to\mathcal{L}_g^{1\pm2+\pm}(\Gamma_1-\tilde{\Gamma}_1,\Gamma_2-\tilde{\Gamma}_2,c_1-\tilde{c}_1,c_2-\tilde{c}_2,\bar{c}_1-\tilde{\bar{c}}_1,\\&\quad c_2-\tilde{\bar{c}}_2,b_1-\tilde{b}_1,b_2-\tilde{b}_2)+\mathcal{L}^{1\pm2+\pm}(\Gamma_1-\tilde{\Gamma}_1,\Gamma_2-\tilde{\Gamma}_2).\end{aligned} \tag{14}$$

Here the fields $\tilde{\Gamma}_{1a},\tilde{\Gamma}_{2a},\tilde{c}_1,\tilde{c}_2,\tilde{\bar{c}}_1,\tilde{\bar{c}}_2,\tilde{b}_1,\tilde{b}_2$ are quantum fluctuations around the background fields. Let us express the fields as a sum of the background fields and quantum fluctuations around them,

$$\begin{aligned}&\Gamma_{1a}\to\Gamma_{1a}+\tilde{\Gamma}_{1a},\Gamma_{2a}\to\Gamma_{2a}+\tilde{\Gamma}_{2a},\\&c_1\to c_1+\tilde{c}_1,c_2\to c_2+\tilde{c}_2,\\&\bar{c}_1\to\bar{c}_1+\tilde{\bar{c}}_1,\bar{c}_2\to\bar{c}_2+\tilde{\bar{c}}_2,\\&b_1\to b_1+\tilde{b}_1,b_2\to b_2+\tilde{b}_2.\end{aligned} \tag{15}$$

So, the covariant derivative transforms to $\nabla_{1a}\to\bar{\nabla}_{1a}=D_a-i\Gamma_{1a}-i\tilde{\Gamma}_{1a}$ and $\nabla_{2a}\to\bar{\nabla}_{2a}=D_a-i\Gamma_{2a}-i\tilde{\Gamma}_{2a}$ Now the quantum fluctuations transform as follows:

$$\begin{aligned}&s_b\,\tilde{\Gamma}_{1a}=\psi_{1a}-\bar{\nabla}_{1a}(c_1-\tilde{c}_1),\,s_b\,\tilde{\Gamma}_{2a}=\psi_{2a}-\bar{\nabla}_{2a}(c_2-\tilde{c}_2),\\&s_b\,\tilde{c}_1=\lambda_1-\frac{1}{2}[c_1-\tilde{c}_1,c_1-\tilde{c}_1],\\&s_b\,\tilde{c}_2=\lambda_2-\frac{1}{2}[c_2-\tilde{c}_2,c_2-\tilde{c}_2],\\&s_b\,\tilde{\bar{c}}_1=\bar{\lambda}_1-(b_1-\tilde{b}_1),\,s_b\,\tilde{\bar{c}}_2=\bar{\lambda}_2-(b_2-\tilde{b}_2),\\&s_b\,\tilde{b}_1=\mu_1,\,s_b\,\tilde{b}_2=\mu_2.\end{aligned} \tag{16}$$

The BRST symmetry for the background fields can be expressed as

$$\begin{aligned}&s_b\,\Gamma_{1a}=\psi_{1a},\,s_b\,\Gamma_{2a}=\psi_{2a},\\&s_b\,c_1=\lambda_{1a},\,s_b\,c_2=\lambda_{2a},\\&s_b\,\bar{c}_1=\bar{\lambda}_1,\,s_b\,\bar{c}_2=\bar{\lambda}_2,\\&s_b\,b_1=\mu_1,\,s_b\,b_2=\mu_2.\end{aligned} \tag{17}$$

Here we have introduced new ghosts associated with the shift symmetry, and the BRST transformation of these new ghost fields vanishes $s_b\psi_{1a}=s_b\psi_{2a}=0$, and $s_b\lambda_1=s_b\lambda_2=s_b\bar{\lambda}_1=s_b\bar{\lambda}_2=s_b\mu_1=s_b\bar{\mu}_2=0$. Now we double the field content of this theory by adding a set of anti-fields corresponding to each field, and the BRST transformation of these anti-fields is given by

$$\begin{aligned}&s_b\,\Gamma^*_{1a}=u_{1a},\,s_b\,\Gamma^*_{2a}=u_{2a},\\&s_b\,c_1^*=v_1,\,s_b\,{c_2}^*=v_2,\\&s_b\,\bar{c}_1^*=\bar{v}_1,\,s_b\,\bar{c}_2^*=\bar{v}_2,\\&s_b\,b_1^*=t_1,\,s_b\,b_2^*=t_2.\end{aligned} \tag{18}$$

Finally, the BRST transformation of these auxiliary fields vanish, $s_bu_{1a}=s_bu_{2a}=0$ and $s_bv_1=s_bv_2=s_b\bar{v}_1=s_b\bar{v}_2=s_bt_1=s_bt_2=0$.

Now we can add the following term to the sum of the gauge fixing term and ghost term:

$$\begin{aligned}\mathcal{L}_f^{1+2+}&=(D_1^2+\partial_3)\left[\Gamma^{1a*}s_b\Gamma_{1a}-c_1^*s_bc_1\right]_{\theta_1=0}\\&\quad+(D_2^2+\partial_3)\left[\Gamma^{2a*}s_b\Gamma_{2a}-c_2^*s_bc_2\right]_{\theta_2=0},\\\mathcal{L}_f^{1-2-}&=(D_1^2-\partial_3)\left[\Gamma^{1a*}s_b\Gamma_{1a}-c_1^*s_bc_1\right]_{\theta_1=0}\\&\quad+(D_2^2-\partial_3)\left[\Gamma^{2a*}s_b\Gamma_{2a}-c_2^*s_bc_2\right]_{\theta_2=0},\\\mathcal{L}_f^{1+2-}&=(D_1^2+\partial_3)\left[\Gamma^{1a*}s_b\Gamma_{1a}-c_1^*s_bc_1\right]_{\theta_1=0}\\&\quad+(D_2^2-\partial_3)\left[\Gamma^{2a*}s_b\Gamma_{2a}-c_2^*s_bc_2\right]_{\theta_2=0},\\\mathcal{L}_f^{1-2+}&=(D_1^2-\partial_3)\left[\Gamma^{1a*}s_b\Gamma_{1a}-c_1^*s_bc_1\right]_{\theta_1=0}\\&\quad+(D_2^2+\partial_3)\left[\Gamma^{2a*}s_b\Gamma_{2a}-c_2^*s_bc_2\right]_{\theta_2=0}.\end{aligned} \tag{19}$$

Now we can write the total action for this theory as

$$\Gamma^{1+2+0}=\int \mathrm{d}^3x\left[\mathcal{L}^{1\pm2\pm}+\mathcal{L}_g^{1\pm2\pm}+\mathcal{L}_f^{1\pm2\pm}\right]. \tag{20}$$

Then we can calculate the effective action, and to the first order term that corresponds to this classical action. We can write the Slavnov–Taylor identity for this theory as

$$\begin{aligned}&\int \mathrm{d}^3x(D_1^2+\partial_3)\\&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{1a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_1^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_1}+b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0}\\&+\int d^3x(D_2^2+\partial_3)\\&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{2a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_2^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_2}+b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0}\\&=0,\end{aligned}$$

$$\begin{aligned}&\int \mathrm{d}^3x(D_1^2-\partial_3)\\&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{1a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_1^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_1}+b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0}\\&+\int d^3x(D_2^2-\partial_3)\\&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{2a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_2^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_2}+b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0}\\&=0,\end{aligned}$$

$$\begin{aligned}&\int d^3x(D_1^2-\partial_3)\\&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{1a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_1^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_1}+b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0}\end{aligned}$$

$$+\int d^3x(D_2^2+\partial_3)\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{2a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_2^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_2}+b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0}=0,$$

$$\int d^3x(D_1^2+\partial_3)\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{1a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_1^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_1}+b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0}+\int d^3x(D_2^2-\partial_3)\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{2a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c_2^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_2}+b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0}=0. \quad (21)$$

This procedure can be followed by using the effective action to obtain Slavnov–Taylor identity at higher order. It may be noted that the tree level Slavnov–Taylor identity can be used to relate relating the two, three and four point functions. This has been used for analyzing the consistency of occurring at one loop in noncommutative gauge theories [45]. It will be possible to use a similar analysis here and analyze the divergences occurring in the supersymmetric Yang–Mills theory. However, the most important observation of this analysis is that the standard form of the Slavnov–Taylor identity does not get deformed, and it is only the measure that is deformed for such theories. This Slavnov–Taylor identity depend on the gauge symmetry of the theory, and the gauge symmetry of the theory is not broken in Yang–Mills theory by the presence of a boundary.

4 Boundary action

In this section, we will analyze the boundary action by using the projection operators, $P_\pm=(1\pm\gamma^3)/2$. We can project the superderivatives using these projection operators as, $D_{1\pm a}=(P_\pm)_a^{\ b}D_{1b}$ and $D_{2\pm a}=(P_\pm)_a^{\ b}D_{2b}$. The supercharges can also be projected as $Q_{1\pm a}=(P_\pm)_a^{\ b}Q_{1b}$ and $Q_{2\pm a}=(P_\pm)_a^{\ b}Q_{2b}$ [38]. The bulk supercharges Q_{1a} and Q_{2a} can now be expressed as [39]

$$\epsilon^{1a}Q_{1a}=\epsilon^{1a}(P_-+P_+)Q_{1a}=\epsilon^{1+}Q_{1-}+\epsilon^{1-}Q_{1+},$$

$$\epsilon^{2a}Q_{2a}=\epsilon^{2a}(P_-+P_+)Q_{2a}=\epsilon^{2+}Q_{2-}+\epsilon^{2-}Q_{2+}. \quad (22)$$

These bulk supercharges $Q_{1\pm}$, $Q_{2\pm}$, are related to the boundary supercharges $Q'_{1\pm}$, $Q'_{2\pm}$, as

$$Q_{1-}=Q'_{1-}+\theta_{1-}\partial_3,\quad Q_{1+}=Q'_{1+}-\theta_{1+}\partial_3,$$

$$Q_{2-}=Q'_{2-}+\theta_{2-}\partial_3,\quad Q_{2+}=Q'_{2+}-\theta_{2+}\partial_3. \quad (23)$$

Here the boundary supercharges are defined as

$$Q'_{1+}=\partial_{1+}-\gamma^s\theta_{1-}\partial_s,\quad Q'_{1-}=\partial_{1-}-\gamma^s\theta_{1+}\partial_s,$$

$$Q'_{2+}=\partial_{2+}-\gamma^s\theta_{2-}\partial_s,\quad Q'_{2-}=\partial_{2-}-\gamma^s\theta_{2+}\partial_s, \quad (24)$$

where s is the index for the coordinates along the boundary, i.e., the case $\mu=3$ has been excluded for a boundary fixed at x_3. The supercharges $Q_{1\pm}$ and $Q_{2\pm}$ are the generators of the half supersymmetry for the bulk fields. Furthermore, $Q'_{1\pm}$ and $Q'_{2\pm}$ are the standard generators of the supersymmetry for the boundary fields. It is possible to express the boundary supercharges as [42]

$$Q'_{1-}=\exp(+\theta_{1+}\theta_{1-}\partial_3)Q_{1-}\exp(-\theta_{1+}\theta_{1-}\partial_3),$$

$$Q'_{1+}=\exp(-\theta_{1-}\theta_{1+}\partial_3)Q_{1+}\exp(+\theta_{1-}\theta_{1+}\partial_3),$$

$$Q'_{2-}=\exp(+\theta_{2+}\theta_{2-}\partial_3)Q_{2-}\exp(-\theta_{2+}\theta_{2-}\partial_3),$$

$$Q'_{2+}=\exp(-\theta_{2-}\theta_{2+}\partial_3)Q_{2+}\exp(+\theta_{2-}\theta_{2+}\partial_3). \quad (25)$$

It is also possible to write the super-algebra of the bulk supercharges in the presence of a boundary as

$$\{Q_{1+a},Q_{1+b}\}=2(\gamma^s_{ab}P_+)\partial_s,\quad \{D_{1+a},D_{1+b}\}=-2(\gamma^s_{ab}P_+)\partial_s,$$
$$\{Q_{1-a},Q_{1-b}\}=2(\gamma^s_{ab}P_-)\partial_s,\quad \{D_{1-a},D_{1-b}\}=-2(\gamma^s_{ab}P_-)\partial_s,$$
$$\{Q_{1+a},Q_{1-b}\}=-2(P_-)_{ab}\partial_3,\quad \{D_{1+a},D_{1-b}\}=2(P_-)_{ab}\partial_3,$$
$$\{Q_{2+a},Q_{2+b}\}=2(\gamma^s_{ab}P_+)\partial_s,\quad \{D_{2+a},D_{2+b}\}=-2(\gamma^s_{ab}P_+)\partial_s,$$
$$\{Q_{2-a},Q_{2-b}\}=2(\gamma^s_{ab}P_-)\partial_s,\quad \{D_{2-a},D_{2-b}\}=-2(\gamma^s_{ab}P_-)\partial_s,$$

$$\{Q_{2+a},Q_{2-b}\}=-2(P_-)_{ab}\partial_3,\quad \{D_{2+a},D_{2-b}\}=2(P_-)_{ab}\partial_3. \quad (26)$$

It may be noted that $\{Q_{1\pm},Q_{2\pm}\}=\{D_{1\pm},D_{2\pm}\}=0$, and $\{Q_{1\pm},D_{2\pm}\}=\{Q_{1\pm},D_{1\pm}\}=\{Q_{2\pm},D_{2\pm}\}=\{Q_{2\pm},D_{1\pm}\}=0$. Thus, we can write

$$D_{1-a}D_{1+b}=(P_-)_{ab}(\partial_3-D_1^2),$$
$$D_{1+a}D_{1-b}=-(P_-)_{ab}(\partial_3+D_1^2),$$
$$D_{2-a}D_{2+b}=(P_-)_{ab}(\partial_3-D_2^2),$$
$$D_{2+a}D_{2-b}=-(P_-)_{ab}(\partial_3+D_2^2). \quad (27)$$

Contracting these equation and using $(P_-)_a^a=1$, we obtained [42]

$$D_1^2+\partial_3=D_{1+}D_{1-},\quad D_2^2+\partial_3=D_{2+}D_{2-}, \quad (28)$$
$$D_1^2-\partial_3=D_{1-}D_{1+},\quad D_2^2-\partial_3=D_{2-}D_{2+}. \quad (29)$$

We can write the Lagrangian for the super-Yang–Mills theory in presence of a boundary as

$$\mathcal{L}^{1+2+}=D_{1+}D_{1-}[W_1^aW_{1a}]_{\theta_1=0}+D_{2+}D_{2-}[W_2^aW_{2a}]_{\theta_2=0},$$

$$\mathcal{L}^{1-2-}=D_{1-}D_{1+}[W_1^aW_{1a}]_{\theta_1=0}+D_{2-}D_{2+}[W_2^aW_{2a}]_{\theta_2=0},$$

$$\mathcal{L}^{1+2-} = D_{1-}D_{1+}[W_1^a W_{1a}]_{\theta_1=0} + D_{2+}D_{2-}[W_2^a W_{2a}]_{\theta_2=0},$$
$$\mathcal{L}^{1-2+} = D_{1+}D_{1-}[W_1^a W_{1a}]_{\theta_1=0} + D_{2-}D_{2+}[W_2^a W_{2a}]_{\theta_2=0}. \tag{30}$$

We can now write the gauge fixing terms in the Lorenz gauge as

$$\begin{aligned}
\mathcal{L}_{gf}^{1+2+} &= D_{1+}D_{1-}\left[b_1(D_1^a\Gamma_{1a}) + \frac{\alpha}{2}b_1^2\right]_{\theta_1=0} \\
&\quad + D_{2+}D_{2-}\left[b_2(D_2^a\Gamma_{2a}) + \frac{\alpha}{2}b^2\right]_{\theta_2=0}, \\
\mathcal{L}_{gf}^{1-2-} &= D_{1-}D_{1+}\left[b_1(D_1^a\Gamma_{1a}) + \frac{\alpha}{2}b_1^2\right]_{\theta_1=0} \\
&\quad + D_{2-}D_{2+}\left[b_2(D_2^a\Gamma_{2a}) + \frac{\alpha}{2}b^2\right]_{\theta_2=0}, \\
\mathcal{L}_{gf}^{1+2-} &= D_{1-}D_{1+}\left[b_1(D_1^a\Gamma_{1a}) + \frac{\alpha}{2}b_1^2\right]_{\theta_1=0} \\
&\quad + D_{2+}D_{2-}\left[b_2(D_2^a\Gamma_{2a}) + \frac{\alpha}{2}b^2\right]_{\theta_2=0}, \\
\mathcal{L}_{gf}^{1-2+} &= D_{1+}D_{1-}\left[b_1(D_1^a\Gamma_{1a}) + \frac{\alpha}{2}b_1^2\right]_{\theta_1=0} \\
&\quad + D_{2-}D_{2+}\left[b_2(D_2^a\Gamma_{2a}) + \frac{\alpha}{2}b^2\right]_{\theta_2=0}.
\end{aligned} \tag{31}$$

The ghost terms corresponding to this gauge fixing term can be written as

$$\begin{aligned}
\mathcal{L}_{gh}^{1+2+} &= D_{1+}D_{1-}\left[\bar{c}_1 D_1^a\nabla_{1a}c_1\right]_{\theta_1=0} \\
&\quad + D_{2+}D_{2-}\left[\bar{c}_2 D_2^a\nabla_{2a}c_2\right]_{\theta_2=0}, \\
\mathcal{L}_{gh}^{1-2-} &= D_{1-}D_{1+}\left[\bar{c}_1 D_1^a\nabla_{1a}c_1\right]_{\theta_1=0} \\
&\quad + D_{2-}D_{2+}\left[\bar{c}_2 D_2^a\nabla_{2a}c_2\right]_{\theta_2=0}, \\
\mathcal{L}_{gh}^{1+2-} &= D_{1-}D_{1+}\left[\bar{c}_1 D_1^a\nabla_{1a}c_1\right]_{\theta_1=0} \\
&\quad + D_{2+}D_{2-}\left[\bar{c}_2 D_2^a\nabla_{2a}c_2\right]_{\theta_2=0}, \\
\mathcal{L}_{gh}^{1-2+} &= D_{1+}D_{1-}\left[\bar{c}_1 D_1^a\nabla_{1a}c_1\right]_{\theta_1=0} \\
&\quad + D_{2-}D_{2+}\left[\bar{c}_2 D_2^a\nabla_{2a}c_2\right]_{\theta_2=0}.
\end{aligned} \tag{32}$$

The total effective Lagrangian which is given by a sum of the gauge fixing term and the ghost term with the original Lagrangian can be written as

$$\begin{aligned}
\mathcal{L}^{1+2+} + \mathcal{L}_g^{1+2+} &= s_b D_{1+}D_{1-}[\bar{c}_1 D_1^a\Gamma_{1a} + \frac{\alpha}{2}\bar{c}_1 b_1]_{\theta_1=0} \\
&+ s_b D_{2+}D_{2-}[\bar{c}_2 D_2^a\Gamma_{2a} + \frac{\alpha}{2}\bar{c}_2 b_2]_{\theta_2=0} \\
&+ D_{1+}D_{1-}[W_1^a W_{1a}]_{\theta_1=0} + D_{2+}D_{2-}[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1-2-} + \mathcal{L}_g^{1-2-} &= s_b D_{1-}D_{1+}[\bar{c}_1 D_1^a\Gamma_{1a} + \frac{\alpha}{2}\bar{c}_1 b_1]_{\theta_1=0} \\
&+ s_b D_{2-}D_{2+}[\bar{c}_2 D_2^a\Gamma_{2a} + \frac{\alpha}{2}\bar{c}_2 b_2]_{\theta_2=0} \\
&+ D_{1-}D_{1+}[W_1^a W_{1a}]_{\theta_1=0} + D_{2-}D_{2+}[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1+2-} + \mathcal{L}_g^{1+2-} &= s_b D_{1-}D_{1+}[\bar{c}_1 D_1^a\Gamma_{1a} + \frac{\alpha}{2}\bar{c}_1 b_1]_{\theta_1=0} \\
&+ s_b D_{2+}D_{2-}[\bar{c}_2 D_2^a\Gamma_{2a} + \frac{\alpha}{2}\bar{c}_2 b_2]_{\theta_2=0} \\
&+ D_{1-}D_{1+}[W_1^a W_{1a}]_{\theta_1=0} + D_{2+}D_{2-}[W_2^a W_{2a}]_{\theta_2=0}, \\
\mathcal{L}^{1-2+} + \mathcal{L}_g^{1-2+} &= s_b D_{1+}D_{1-}[\bar{c}_1 D_1^a\Gamma_{1a} + \frac{\alpha}{2}\bar{c}_1 b_1]_{\theta_1=0} \\
&+ s_b D_{2-}D_{2+}[\bar{c}_2 D_2^a\Gamma_{2a} + \frac{\alpha}{2}\bar{c}_2 b_2]_{\theta_2=0} \\
&+ D_{1+}D_{1-}[W_1^a W_{1a}]_{\theta_1=0} + D_{2-}D_{2+}[W_2^a W_{2a}]_{\theta_2=0}.
\end{aligned} \tag{33}$$

Now we can write $\mathcal{L}_f^{1\pm2\pm}$ as,

$$\begin{aligned}
\mathcal{L}_f^{1+2+} &= D_{1+}D_{1-}[\Gamma^{1a*}s_b\Gamma_{1a} - c_1^* s_b c_1]_{\theta_1=0} \\
&\quad + D_{2+}D_{2-}[\Gamma^{2a*}s_b\Gamma_{2a} - c_2^* s_b c_2]_{\theta_2=0}, \\
\mathcal{L}_f^{1-2-} &= D_{1-}D_{1+}[\Gamma^{1a*}s_b\Gamma_{1a} - c_1^* s_b c_1]_{\theta_1=0} \\
&\quad + D_{2-}D_{2+}[\Gamma^{2a*}s_b\Gamma_{2a} - c_2^* s_b c_2]_{\theta_2=0}, \\
\mathcal{L}_f^{1+2-} &= D_{1+}D_{1-}[\Gamma^{1a*}s_b\Gamma_{1a} - c_1^* s_b c_1]_{\theta_1=0} \\
&\quad + D_{2-}D_{2+}[\Gamma^{2a*}s_b\Gamma_{2a} - c_2^* s_b c_2]_{\theta_2=0}, \\
\mathcal{L}_f^{1-2+} &= D_{1-}D_{1}[\Gamma^{1a*}s_b\Gamma_{1a} - c_1^* s_b c_1]_{\theta_1=0} \\
&\quad + D_{2+}D_{2-}[\Gamma^{2a*}s_b\Gamma_{2a} - c_2^* s_b c_2]_{\theta_2=0}.
\end{aligned} \tag{34}$$

So, the Slavnov–Taylor identity for this theory is

$$\begin{aligned}
&\int d^3x\, D_{1+}D_{1-} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}^*}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}} + \frac{\delta\Gamma^{1+2+0}}{\delta c_1^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_1} + b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0} \\
&+ \int d^3x\, D_{2+}D_{2-} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}^*}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}} + \frac{\delta\Gamma^{1+2+0}}{\delta c_2^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_2} + b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0} \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
&\int d^3x\, D_{1-}D_{1+} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}^*}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}} + \frac{\delta\Gamma^{1+2+0}}{\delta c_1^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_1} + b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0} \\
&+ \int d^3x\, D_{2-}D_{2+} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}^*}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}} + \frac{\delta\Gamma^{1+2+0}}{\delta c_2^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_2} + b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0} \\
&= 0,
\end{aligned}$$

$$\begin{aligned}
&\int d^3x\, D_{1-}D_{1+} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}^*}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}} + \frac{\delta\Gamma^{1+2+0}}{\delta c_1^*}\frac{\delta\Gamma^{1+2+0}}{\delta c_1} + b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0}
\end{aligned}$$

$$
\begin{aligned}
&+\int d^3x\, D_{2+} D_{2-} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{2a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c^*_2}\frac{\delta\Gamma^{1+2+0}}{\delta c_2}+b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0} \\
&=0,
\end{aligned}
$$

$$
\begin{aligned}
&\int d^3x\, D_{1+} D_{1-} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{1a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{1a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c^*_1}\frac{\delta\Gamma^{1+2+0}}{\delta c_1}+b_1\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_1}\right]_{\theta_1=0} \\
&+\int d^3x\, D_{2-} D_{2+} \\
&\times\left[\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma^*_{2a}}\frac{\delta\Gamma^{1+2+0}}{\delta\Gamma_{2a}}+\frac{\delta\Gamma^{1+2+0}}{\delta c^*_2}\frac{\delta\Gamma^{1+2+0}}{\delta c_2}+b_2\frac{\delta\Gamma^{1+2+0}}{\delta\bar{c}_2}\right]_{\theta_2=0} \\
&=0. \qquad (35)
\end{aligned}
$$

It may be noted that it is possible to obtain higher order Slavnov–Taylor identity for such theories. In fact, this procedure can be used to obtain a Slavnov–Taylor identity for any gauge theory in the presence of a boundary. This identity can be used to relate different correlation functions to each other. Thus, they can be used to analyze scattering processes in this theory. It is important to note that this identity preserves only half of the supersymmetry of the original theory.

5 Conclusion

In this paper, we analyzed a three dimensional supersymmetric theory with $\mathcal{N} = 2$ supersymmetry. Even though the BRST symmetry has been analyzed for a Yang–Mills theory with a boundary in $\mathcal{N} = 1$ superspace [43], in this paper, we analyze the BRST symmetry for a Yang–Mills theory with a boundary in $\mathcal{N} = 2$ superspace. The effective Lagrangian was obtained by the sum of the gauge fixing term and the ghost term with the original classical Lagrangian. It was demonstrated that even though the supersymmetry of the effective Lagrangian was broken by the presence of the boundaries, it was possible to preserve half the supersymmetry of this effective Lagrangian This was done by adding new boundary terms to the original bulk effective Lagrangian. The supersymmetric variation of the original bulk effective Lagrangian was exactly canceled by the supersymmetric variation of this new boundary term. Thus, it was possible to retain half of the supersymmetry of this original theory in the presence of a boundary. We also obtain the Slavnov–Taylor identity for this theory.

It may be noted that in the Horava–Witten theory, one of the low energy limits of the heterotic string theory can be obtained from the 11 dimensional supergravity in the presence of a boundary [46–49]. It has been possible in this construction to obtain a unification of gauge and gravitational couplings. Motivated by the original Horava–Witten theory, a five dimensional globally supersymmetric Yang–Mills theory coupled to a four dimensional hypermultiplet on the boundary has already been constructed [50]. It would be interesting to use results of this paper to analyze such a system. It would also be interesting to analyze the BRST symmetry of such a system using both linear and non-linear gauges. Furthermore, the BRST symmetry and gauge fixing have been studied for perturbative quantum gravity [51–56]. It is possible to generalize this work to supergravity solutions, and analyze the supersymmetry of such supergravity solutions, when there is a boundary. In fact, the supergravity solutions with a boundary term have been studied, and this was done using a similar off-shell formalism [57]. It would be interesting to analyze the BRST symmetry for such supergravity theories with a boundary term.

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References

1. C. Becchi, A. Rouet, R. Stora, Ann. Phys. **98**, 287 (1976)
2. I.V. Tyutin, Lebedev preprint fian. 39 (1975). arXiv:0812.0580
3. D. Dudal, H. Verschelde, V.E.R. Lemes, M.S. Sarandy, S.P. Sorella, M. Picariello, Ann. Phys. **308**, 62 (2003)
4. R. Delbourgo, P.D. Jarvis, J. Phys. A Math. Gen. **15**, 611 (1982)
5. M. Faizal, S. Upadhyay, Phys. Lett. B **736**, 288 (2014)
6. L. Baulieu, J. Thierry-Mieg, Nucl. Phys. B **197**, 477 (1982)
7. D. Dudal, V.E.R. Lemes, M. Picariello, M.S. Sarandy, S.P. Sorella, H. Verschelde, JHEP **0212**, 008 (2002)
8. M. Faizal, D.J. Smith, Phys. Rev. D **85**, 105007 (2012)
9. M. Faizal, Phys. Rev. D **84**, 106011 (2011)
10. M. Faizal, Commun. Theor. Phys. **57**, 637 (2012)
11. M. Faizal, Mod. Phys. Lett. A **27**, 1250147 (2012)
12. S. Upadhyay, D. Das, Phys. Lett. B **733**, 63 (2014)
13. K.I. Kondo, arXiv:hep-th/0103141
14. D. Dudal, H. Verschelde, V.E.R. Lemes, M.S. Sarandy, S.P. Sorella, M. Picariello, A. Vicini, J.A. Gracey, JHEP **0306**, 003 (2003)
15. D. Dudal, H. Verschelde, J. Phys. A **36**, 8507 (2003)
16. V.E.R. Lemes, M.S. Sarandy, S.P. Sorella, Ann. Phys. **308**, 1 (2003)
17. A.R. Fazio, V.E.R. Lemes, M. Picariello, M.S. Sarandy, S.P. Sorella, Mod. Phys. Lett. A **18**, 711 (2003)
18. M. Schaden, arXiv:hep-th/9909011
19. K.-I. Kondo, T. Shinohara, Phys. Lett. B **491**, 263 (2000)
20. G. 't Hooft, Nucl. Phys. B **190**, 455 (1981)
21. Y. Nambu, Phys. Rev. D **10**, 4262 (1974)
22. S. Mandelstam, Phys. Rep. **23**, 245 (1976)
23. A.M. Polyakov, Nucl. Phys. B **120**, 429 (1977)
24. K.I. Kondo, Phys. Rev. D **58**, 085013 (1998)
25. K.I. Kondo, Phys. Rev. D **58**, 105016 (1998)
26. K.I. Kondo, Phys. Lett. B **455**, 251 (1999)
27. L. Baulieu, S.P. Sorella, Phys. Lett. B **671**, 481 (2009)
28. L. Baulieu, M.A.L. Capri, A.J. Gomez, V.E.R. Lemes, R.F. Sobreiro, S.P. Sorella, Eur. Phys. J. C **66**, 451 (2010)
29. D. Dudal, S.P. Sorella, N. Vandersickel, H. Verschelde, Phys. Rev. D **79**, 121701 (2009)

30. P.M. Lavrov, O.V. Radchenko, A.A. Reshetnyak, Mod. Phys. Lett. A. **27**, 1250067 (2012)
31. D.V. Belyaev, JHEP **0601**, 046 (2006)
32. D.V. Belyaev, JHEP **0601**, 047 (2006)
33. P. van Nieuwenhuizen, D.V. Vassilevich, Class. Quantum Gravity **22**, 5029 (2005)
34. U. Lindstrom, M. Rocek, P. van Nieuwenhuizen, Nucl. Phys. B **662**, 147 (2003)
35. P. Di Vecchia, B. Durhuus, P. Olesen, J.L. Petersen, Nucl. Phys. B **207**, 77 (1982)
36. P. Di Vecchia, B. Durhuus, P. Olesen, J.L. Petersen, Nucl. Phys. B **217**, 395 (1983)
37. Y. Igarashi, Phys. Rev. D **30**, 1812 (1984)
38. D.V. Belyaev, P. van Nieuwenhuizen, JHEP **0804**, 008 (2008)
39. D.S. Berman, D.C. Thompson, Nucl. Phys. B **820**, 503 (2009)
40. M. Faizal, Mod. Phys. Lett. A **29**, 1450154 (2014)
41. M. Faizal, JHEP **1204**, 017 (2012)
42. M. Faizal, D.J. Smith, Phys. Rev. D **87**, 025019 (2013)
43. M. Faizal, Int. J. Theor. Phys. **52**, 392 (2013)
44. S.J. Gates Jr., M.T. Grisaru, M. Rocek, W. Siegel, Front. Phys. **58**, 1 (1983)
45. D.N. Blaschke, H. Grosse, J.C. Wallet, JHEP **1306**, 038 (2013)
46. P. Horava, E. Witten, Nucl. Phys. B **475**, 94 (1996)
47. E. Witten, Nucl. Phys. B **471**, 135 (1996)
48. P. Horava, Phys. Rev. D **54**, 7561 (1996)
49. P. Horava, E. Witten, Nucl. Phys. B **460**, 506 (1996)
50. E.A. Mirabelli, M.E. Peskin, Phys. Rev. D **58**, 065002 (1998)
51. M. Faizal, Found. Phys. **41**, 270 (2011)
52. G. Esposito, G. Fucci, A.Y. Kamenshchik, K. Kirsten, Class. Quantum Gravity **22**, 957 (2005)
53. G. Esposito, AYu. Kamenshchik, I.V. Mishakov, G. Pollifrone, Phys. Rev. D **52**, 3457 (1995)
54. M. Faizal, Class. Quantum Gravity **29**, 035007 (2012)
55. M. Faizal, Mod. Phys. Lett. A **28**, 1350034 (2013)
56. M. Faizal, J. Phys. A **44**, 402001 (2011)
57. D.V. Belyaev, P. van Nieuwenhuizen, JHEP **0809**, 069 (2008)